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Inverse Kinematic Modeling of A 3-Axes Planar Articulated Robotic Arm (PLANBOT)

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Abstract : This paper presents the inverse kinematic analysis and the modeling of a indigenously developed 3-axes planar articulated robot arm with indigenous components and to use that developed inverse kinematic model to perform a glass cutting application in the operational space of the planar arm with a diamond cutter / drill bit as the tool at the end-effector point. A user friendly GUI developed in Matlab is used for entering the inputs to the system. The main objective of this work was to obtain the general-purpose inverse kinematic equations, so that those equations can be used by any 3-axes planar mechanisms to do any type of job / application.

Keywords : Planar mechanisms, Inverse kinematics, Drilling operation, Cutting operation, Tool configuration vector, Row operations**.**

I. INTRODUCTION

Robotics is an interdisciplinary field that mixes various engineering disciplines into one. A modest attempt was made in this field to design and fabricate a unique 3-axes system with indigenous components. For this developed and fabricated system, a mathematical model based on the inverse kinematics was obtained. The main objective of this research work was to develop a inverse kinematic mathematical model for a indigenously developed 3-axes planar articulated robot arm, known as 'PLANBOT' and to use that mathematical model to do a successful pick and place task or to do some application such as a drilling operation or a glass cutting operation.

A 3-axes planar articulated robot arm is designed and fabricated in the college laboratory. A user friendly GUI developed in MATLAB language is used for maneuvering the planar mechanism, which we have named it as the **PLAN**ar ro**BOT**. This user friendly GUI is interconnected to the hardware system through the computer, controller, electronic circuitries and the software. This user-friendly module is designed and successfully implemented for the operation of the entire system so as to do some specific operation, say cutting a glass with a diamond cutter placed at the tip of the planbot which acts as the tool.

The mechanical hardware of the robotic system consists of 3 actuators, 2 links & 3 joints. All the 3 joints are rotary or articulated in nature. The 1st joint is named as the base, the 2nd one is named as the shoulder & the 3rd joint is named as the roll to which a diamond cutter / drill bit is attached to cut the glass / drill a hole. Thus, the hardware system is having 3 DOF, with 2 positioning axes and 1 orientation axis in a plane. Stepper motors actuate all the 3 joints. The planar robot was also simulated on the computer & the successful demonstration of the simulation results & the experimental results were also validated using the IK model.

The pick and place point (in 2D) in the work space of the planbot is given as input to the computer which in turn is connected to the hardware using the parallel port and the driver, i.e., the controller. Once, the input pick and place point is given to the computer, this input is taken by the developed mathematical inverse kinematic model. The software and this IK model calculates the sets of joint variables to go and reach that pick and place point and the robot immediately goes and stops at that point in the shortest path. A brief inverse kinematic modeling was also carried out for the developed 3-axes planar mechanism. Row operations used in trigonometry in mathematics are used to develop the inverse kinematic model, which consists of a set of IK equations [5].

The 2 inputs to the inverse kinematic model are the position - orientation and the geometric link parameters, i.e., the physical dimensions of the robot arm, which is a constant parameter and stored in the memory of the computer. Planbot employs a sophisticated application - controlling interface created in Matlab, as it was a fantastic programming language for any application software development [6].

The inverse kinematic model was successfully demonstrated to do a glass cutting operation. Sensors are also used at the tip of the robot for sensing the actual pressure that is to be applied to cut the glass or to drill a hole in the wood. The tool used for performing this operation was a diamond cutter which was attached to the tip of the roll motor, which was fixed at the end of the planar mechanism which was acting as the tool (EE). The developed inverse kinematic model was also used to perform another operation, i.e., to do a drilling operation in a glass or a wooden sheet for which the tool used is a drill bit. This developed inverse kinematic model also incorporates the obstacle avoiding algorithms also during the pick and place operation / job / application [7].

The paper is organized as follows. First, a introduction to robotics, the physical construction of the planar mechanism was presented in the previous section. In section 2, the block diagram of the inverse kinematic model along with the LCD is presented. Thirdly, the inverse kinematic algorithm is presented in section 3. Section 4 gives the development of the IK modeling equations. Section 5 gives the advantages of the inverse kinematically modeled robot along with the conclusions followed by the references.

II. BLOCK DIAGRAM OF THE IK MODELING

In this section, we develop the inverse kinematic mathematical model for performing any application by the designed and fabricated unit. The 3 axis planar articulated robot arm's LCD is shown in Fig. 1. There are 2 links a_1 and a_2, which are moving parallel to the plane of the work surface, and there are 3 rotary joints J_1, J_2, J_3. All the 3 joint axes are parallel and $\perp^r$ to the work surface x^0 y^0. The 3 joint variables are given by θ_1, θ_2, θ_3. The tool is facing in the downward direction as a result of which the approach vector $r^3 = z^3$ is pointing downwards as shown in the LCD [1].

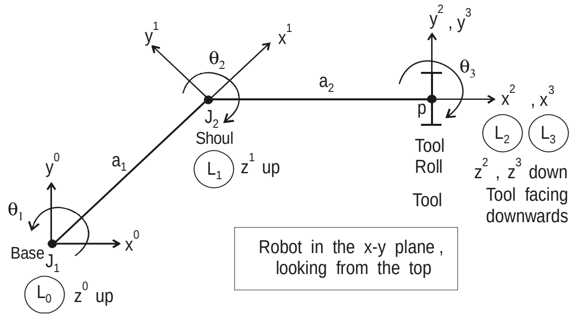

Fig. 1 LCD of the 3-axes planar arm

To develop the IK model, for the one line diagram of the 3-axes arm as shown in the Fig. 1, first, we state the IK postulate as, "*given the geometric link parameters and the position and orientation or the tool configuration vector w, finding the sets of joint variables which will satisfy the same position and orientation*". The link parameters are constant for a given system and are to be given as they are the physical dimensions of the system.

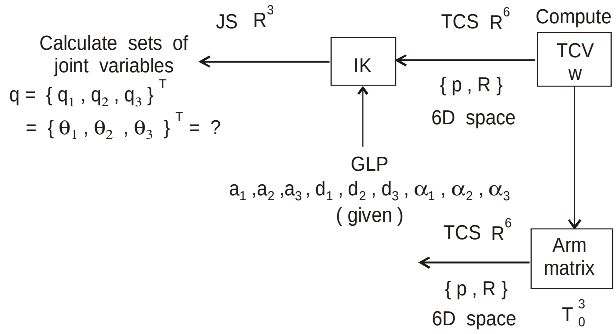

Fig. 2. Inverse Kinematic Block Diagram

III. INVERSE KINEMATIC ANALYSIS ALGO

The inverse kinematic algorithm developed is shown as below [1], [2].

Input the arm matrix T_0^3

↓

Input the GLP a, d, α

↓

Compute the TCV $w(q) = \begin{bmatrix} w^1 \\ w^2 \end{bmatrix}$

↓

Compute $\left(w_1^2 + w_2^2\right)$

↓

Compute shoulder angle

$$q_2 = \pm\cos^{-1}\left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2\,a_1\,a_2}\right]$$

↓

Compute base angle

$$q_1 = \tan^{-1}\left[\frac{(a_1 + a_2C_2)\,w_2 - (a_2S_2)\,w_1}{(a_1 + a_2C_2)\,w_1 + (a_2S_2)\,w_2}\right]$$

↓

Compute global tool roll angle

$$q_{123} = a\tan 2\,(R_{21}, R_{11})$$

↓

Compute tool roll angle

$$q_3 = \pi \ln |w_6|$$

or

$$q_3 = q_{123} - q_2 - q_3$$

Fig. 3. Inverse Kinematic Algorithm

IV. THE DEVELOPMENT OF THE IK MODEL

Output of DK : position p and orientation R of the tool w.r.t. base is given by [1], [3]

$$T_{Base}^{Tool}(q) = T_0^3(q) = \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1C_1 + a_2C_{12} \\ S_{123} & C_{123} & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_0^3 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & p_1 \\ R_{21} & R_{22} & R_{23} & p_2 \\ R_{31} & R_{32} & R_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_0^3 = \begin{bmatrix} r^1 & r^2 & r^3 & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

norm slid app posn

$$p_1 = w_1 = a_1 C_1 + a_2 C_{12} \quad (4)$$

$$p_2 = w_2 = a_1 S_1 + a_2 S_{12} \quad (5)$$

$$p_3 = w_3 = d_3 \quad (6)$$

$$w_4 = w_5 = 0 \quad (7)$$

$$w_6 = \exp\left(\frac{q_3}{\pi}\right) \quad (8)$$

$$R_{11} = R_{22} = C_{123} \quad (9)$$

$$R_{12} = -S_{123} \quad (10)$$

$$R_{21} = S_{123} \quad (11)$$

$$R_{33} = 1\,;\quad R_{13} = R_{23} = 0 \quad (12)$$

The 2 inputs to the IKA is the tool configuration vector, which is obtained from the arm matrix and the other being the geometric link parameters, which is constant for the given system [1], [4].

The outputs of the IKA, being the sets of joint variables, which is used to satisfy the same position and orientation. The tool configuration vector is given by [1]

$$w(q) = \begin{bmatrix} w^1 \\ \cdots\cdots \\ w^2 \end{bmatrix} = \begin{bmatrix} p \\ \cdots\cdots \\ \left\{\exp\left(\frac{q_3}{\pi}\right)\right\} r^3 \end{bmatrix} \quad (13)$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \cdots\cdots \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \cdots\cdots \\ \left\{\exp\left(\frac{q_3}{\pi}\right)\right\} R_{13} \\ \left\{\exp\left(\frac{q_3}{\pi}\right)\right\} R_{23} \\ \left\{\exp\left(\frac{q_3}{\pi}\right)\right\} R_{33} \end{bmatrix} \quad (14)$$

$$w(q) = \begin{bmatrix} a_1C_1 + a_2C_{12} \\ a_1S_1 + a_2S_{12} \\ d_3 \\ \cdots\cdots \\ 0 \\ 0 \\ \exp\left(\frac{q_3}{\pi}\right) \end{bmatrix}_{(6\times 1)} \quad (15)$$

To Extract Shoulder Joint Variable $q_2 = \theta_2$

Squaring and adding w_1 and w_2, we get [5]

$$w_1^2 + w_2^2 = (a_1 C_1 + a_2 C_{12})^2 + (a_1 S_1 + a_2 S_{12})^2$$

$$= a_1^2 + a_2^2 + 2a_1a_2 C_2 \quad (16)$$

$$\therefore, q_2 = \theta_2 = \pm\cos^{-1}\left\{\frac{w_1^2 + w_2^2 - a_1^2 + a_2^2}{2\,a_1\,a_2}\right\} \quad (17)$$

From the above Eq. (17), we see that the IK solution is not unique and hence, we get two solutions given by $q_2 = -$; right handed solution, i.e., < 0 ; link a_2 moves to right [9].

To Extract Shoulder Joint Variable $q_1 = \theta_1$

$$w_1 = a_1 C_1 + a_2 C_{12}$$

$$w_2 = a_1 S_1 + a_2 S_{12}$$

Expand C_{12} and S_{12} using sum of sines and cosines ; isolate C_1, S_1 write in matrix form, collect all C_1 terms and S_1 terms, find A^{-1} and $|A|$, solve for q_1 [1],[6].

$$w_1 = a_1 C_1 + a_2 (C_1 C_2 - S_1 S_2)$$
$$= (a_1 + a_2 C_2) C_1 - (a_2 S_2) S_1 \quad (18)$$
$$w_2 = a_1 S_1 + a_2 (S_1 C_2 + C_1 S_2)$$
$$= (a_2 S_2) C_1 + (a_1 + a_2 C_2) S_1 \quad (19)$$

Writing equations for w_1 and w_2 in matrix form, we get [1]

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 C_2 & -a_2 S_2 \\ a_2 S_2 & a_1 + a_2 C_2 \end{bmatrix} \begin{bmatrix} C_1 \\ S_1 \end{bmatrix} = [A] \begin{bmatrix} C_1 \\ S_1 \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} C_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 C_2 & -a_2 S_2 \\ a_2 S_2 & a_1 + a_2 C_2 \end{bmatrix}^{-1} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = [A]^{-1} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (21)$$

A^{-1} can be computed using $\frac{\text{adj A}}{\text{det A}}$, which is equal to the $\frac{\text{transpose of the co-factors of A}}{\text{determinant of A}}$. Note that for a (2 × 2) matrix, adjoint of A is noting but $A^{\mathbf{T}}$. Determinant of A is given by

$$|A| = (a_1 C_1 + a_2 C_{12})^2 + (a_2 S_2)^2$$
$$= a_1^2 + a_2^2 + 2 a_1 a_2 C_2$$
$$= w_1^2 + w_2^2 \quad (22)$$

Substituting the values of A^{-1} in Eq. (21), we get [1]

$$\begin{bmatrix} C_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} \frac{a_1 + a_2 C_2}{w_1^2 + w_2^2} & \frac{a_2 S_2}{w_1^2 + w_2^2} \\ \frac{-a_2 S_2}{w_1^2 + w_2^2} & \frac{a_1 + a_2 C}{w_1^2 + w_{2\,2}^2} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (23)$$

$$C_1 = \frac{(a_1 + a_2 C_2) w_1 + (a_2 S_2) w_2}{w_1^2 + w_2^2} \quad (24)$$

$$q_1 = \theta_1 = \cos^{-1} \left\{ \frac{(a_1 + a_2 C_2) w_1 + (a_2 S_2) w_2}{w_1^2 + w_2^2} \right\} \quad (25)$$

Similarly,

$$S_1 = \frac{(a_1 + a_2 C_2) w_2 - (a_2 S_2) w_1}{w_1^2 + w_2^2} \quad (26)$$

$$q_1 = \theta_1 = \sin^{-1} \left\{ \frac{(a_1 + a_2 C_2) w_2 - (a_2 S_2) w_1}{w_1^2 + w_2^2} \right\} \quad (27)$$

The above 2 equations (25) & (27) can be used to find the value of q_1. But, it gives the base angles only over the range (–90°, +90°), i.e., 180° range. We want the base angles over the complete range, i.e., 360° range. Therefore, divide S_1 by C_1 to get $\tan q_1$ or $\tan \theta_1$ [1].

$$\tan \theta_1 = \frac{S_1}{C_1} = \left[\frac{(a_1 + a_2 C_2) w_2 - (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 + (a_2 S_2) w_2} \right] \quad (28)$$

$$q_1 = \theta_1 = \tan^{-1} \left[\frac{(a_1 + a_2 C_2) w_2 - (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 + (a_2 S_2) w_2} \right] \quad (29)$$

The solution given by the above 2 equations gives the values of the base angles q_1 over the complete range $(-\pi, +\pi)$, i.e., 360° ; since, we had used the arc tan 2 function.

To Extract Tool Roll Joint Variable $q_3 = \theta_3$

From the TCV, we have ; $w_4 = w_5 = 0$ [1]

$$\therefore, \; q_n = \pi \ln \sqrt{w_4^2 + w_5^2 + w_6^2}$$
$$\therefore, \; q_3 = \pi \ln |w_6| \quad (30)$$

The last joint variable θ_3 can also be calculated from the components of the rotation matrix from Eq. (1) as

$$\frac{R_{21}}{R_{11}} = \frac{S_{123}}{C_{123}} = \tan q_{123} = \tan (\theta_1 + \theta_2 + \theta_3)$$
$$\theta_3 = \theta_{123} - \theta_1 - \theta_2 . \quad (31)$$

Thus, the IK model equations are finally obtained [8].

V. CONCLUSION

A inverse kinematic analysis was performed for the designed 3-axes robot and was successfully implemented using a user friendly software written in C++ GUI. The robot was controlled using the developed GUI in various modes. The task say, the drilling operation's (drill point) position and orientation was given as the input to the IK algo and the algo generates the set of joint angles necessary to go to that drill point and do the drill operation. Similarly, a cutting task was also implemented using a diamond cutter as the tool for cutting the glass in the form of a circle, square, elliptical fashion, etc.,.

REFERENCES

[1]. Robert, J.S., "Fundamentals of Robotics : Analysis and Control", *PHI*, New Delhi., 1992.

[2]. Klafter, Thomas and Negin, "Robotic Engineering", *PHI*, New Delhi, 1990.

[3]. Lovass Nagy V, R.J. Schilling, "Control of Kinematically Redundant Robots Using {1}-inverses", *IEEE Trans. Syst. Man*, *Cybernetics*, Vol. SMC-17, No. 4, pp. 644-649, 1987.

[4]. Fu, Gonzalez and Lee, "Robotics : Control, Sensing, Vision and Intelligence", *McGraw Hill*, Singapore, 1995.

[5]. Ranky, P. G., C. Y. Ho, "Robot Modeling, Control & Applications", *IFS Publishers*, *Springer*, UK., 1998.

[6]. T.C.Manjunath, "Fundamentals of Robotics", *Nandu Publishers*, *5th Revised Edition*, Mumbai., India, 2007.

[7]. Phillip Coiffette, "Robotics Series, Volume I to VIII, *Kogan Page*, London, UK, 1995.

[8]. Luh, C.S.G., M.W. Walker, and R.P.C. Paul, "On-line computational scheme for mechanical manipulators", *Journal of Dynamic Systems*, *Measurement & Control*, Vol. 102, pp. 69-76, 1998.

[9]. Lorenzo and Siciliano, "Modeling and Control of Robotic Manipulators", *McGraw Hill*.

[10]. Carl D Crane and Joseph Duffy, "Kinematic Analysis of Robot Manipulators", *Cambridge Press*, UK.